MATHEMATICS KANGAROO 2011
Austria - 17.3.2011

Group: Student, Grades: 11 onwards

Name:	
School:	
Class:	

Time allowed: 75 min.
Each correct answer, questions 1.-10.: 3 Points
Each correct answer, questions 11.-20.: 4 Points
Each correct answer, questions 21.-30.: 5 Points
Each question with no answer given: 0 Points
Each incorrect answer: Lose ¼ oft he points for that question.
You begin with 30 points.

Please write the letter (A, B, C, D, E) of the correct answer under the question number (1 to 30). Write neatly and carefully!

1	2	3	4	5	6	7	8	9	10

11	12	13	14	15	16	17	18	19	20

21	22	23	24	25	26	27	28	29	30

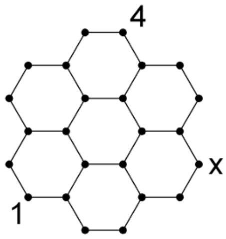
Information über den Känguruwettbewerb: www.kaenguru.at
Wenn Du mehr in dieser Richtung machen möchtest, gibt es die Österreichische Mathematikolympiade; Infos unter: www.oemo.at

Ich melde mich zur Teilnahme zum österreichischen Wettbewerb „Känguru der Mathematik 2011“ an.
Ich stimme zu, dass meine personenbezogenen Daten, nämlich Vor- und Zuname, Geschlecht, Klasse, Schulstufe, Schulstandort und Schulart zum Zweck der Organisation und Durchführung des Wettbewerbs, der Auswertung der Wettbewerbsergebnisse (Ermitteln der erreichten Punkte und Prozentzahlen), des Erstellens von landes- sowie österreichweiten Reihungen, der Veröffentlichung der Ergebnisse jener Schülerinnen und Schüler, die in ihrer Kategorie zumindest 50% der zu vergebenden Punkte erreicht haben sowie des Ermöglichens von Vergleichen mit eigenen Leistungen aus vorherigen Wettbewerbsperioden auf www.kaenguru.at. bzw. http://kaenguru.diefenbach.at/ verwendet werden.
Die Verwendung dieser Daten ist bis 31. Dezember 2013 gestattet. Diese Zustimmung kann ich gemäß § 8 Abs. 1 Z 2 DSG 2000 ohne Begründung jederzeit schriftlich bei webmaster@kaenguru.at widerrufen.
Nach dem 31. Dezember 2013 werden Vor- und Zuname, die Klasse und der Schulstandort gelöscht, wobei das zuletzt genannte Datum durch die Angabe des Bundeslandes ersetzt wird. Die Verwendung der auf diese Art pseudonymisierten Daten ist nur mehr für statistische Zwecke auf der Grundlage von § 46 Abs. 1 Z 3 DSG 2000 erlaubt.

Unterschrift:

Mathematics Kangaroo 2011
Group Student (Grade 11 onwards)
Austria - 17.3.2011

- 3 Point Questions -



1) In the picture on the right a number should be written next to each point. The sum of the numbers on the corners of each side of the hexagon should be equal. Two numbers have already been inserted. Which number should be in the place marked ‘x’?

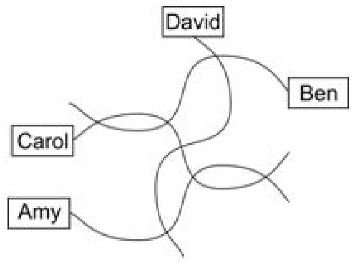
A) 1 **B)** 3 **C)** 4 **D)** 5 **E)** 24

2) Three racers take part in a Formula-1-Race: Michael, Fernando and Sebastian. From the start Michael is in the lead in front of Fernando who is in front of Sebastian. In the course of the race Michael and Fernando overtake each other 9 times, Fernando and Sebastian 10 times and Michael and Sebastian 11 times. In which order do those three end the race?

A) Michael, Fernando, Sebastian **B)** Fernando, Sebastian, Michael **C)** Sebastian, Michael, Fernando **D)** Sebastian, Fernando, Michael **E)** Fernando, Michael, Sebastian

3) If $2^x = 15$ and $15^y = 32$ then xy equals

A) 5 **B)** $\log_2 15 + \log_{15} 32$ **C)** $\log_2 47$ **D)** 7 **E)** $\sqrt{47}$

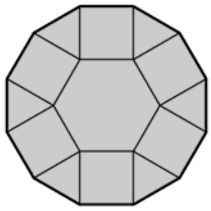


4) Jan cannot draw very accurately but nevertheless he tried to produce a roadmap of his village. The relative position of the houses and the street crossings are all correct but three of the roads are actually straight and only Qurwik street is not. Who lives in Qurwik street?

A) Amy **B)** Ben **C)** Carol **D)** David **E)** It cannot be determined from the drawing.

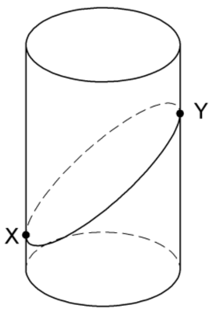
5) All four-digit numbers whose digit sum is 4 are written down in descending order. In which position is the number 2011?

A) 6. **B)** 7. **C)** 8. **D)** 9. **E)** 10.



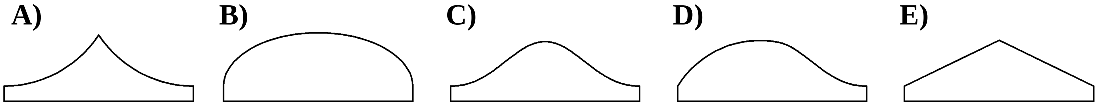
6) Given are a regular hexagon with side-length 1, six squares and six equilateral triangles as shown on the right. How big is the perimeter of this tessellation?

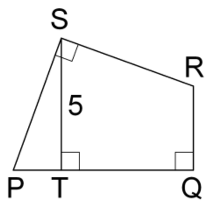
A) $6(1+\sqrt{2})$ **B)** $6\left(1+\frac{\sqrt{3}}{2}\right)$ **C)** 9 **D)** $6+3\sqrt{2}$ **E)** 12



7) A rectangular piece of paper is wrapped around a cylinder. Then an angled straight cut is made through the points X and Y of the cylinder as shown on the left. The lower part of the piece of paper is then unrolled. Which of the following pictures could show the result?

A) **B)** **C)** **D)** **E)**





8) Determine the area of the quadrilateral PQRS pictured on the right, where PS = RS, $\angle PSR = \angle PQR = 90°$, ST ∞ PQ, and ST = 5.

A) 20 **B)** 22.5 **C)** 25 **D)** 27.5 **E)** 30

9) Andrew wrote down all odd numbers from 1 to 2011 on a board. Bob then deleted all multiples of three. How many numbers remained on the board?

A) 335 **B)** 336 **C)** 671 **D)** 1005 **E)** 1006

10) Max and Hugo roll a number of dice in order to decide who has to be the first one to jump into the cold lake. If there is no six, then Max has to jump. If there is one six, then Hugo has to jump and if there are several sixes neither will have to jump in. How many dice do they have to use so that the probability of either of them having to jump in is equal?

A) 3 **B)** 5 **C)** 8 **D)** 9 **E)** 17

- 4 Point Questions -

11) A rectangle is split into three smaller rectangles. One of which has the measurements 7 by 11. Another one has the measurements 4 by 8. Determine the measurements of the third rectangle so that its area is as large as possible.

A) 1 by 11 **B)** 3 by 4 **C)** 3 by 8 **D)** 7 by 8 **E)** 7 by 11

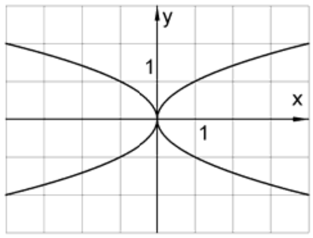
	2	
1		3
	4	

12) Michael wants to write whole numbers into the empty fields of the 3×3 table on the right so that the sum of the numbers in each 2×2 square equals 10. Four numbers have already been written down. Which of the following values could be the sum of the remaining five numbers?

A) 9 **B)** 10 **C)** 12 **D)** 13 **E)** None of these numbers is possible.

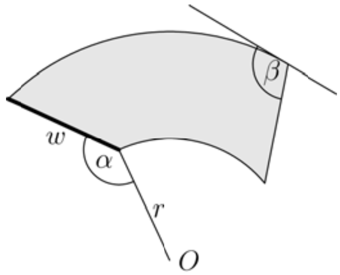
13) 48 children are going on a ski trip. Six of which go with exactly one sibling, nine go with exactly two siblings and four with three siblings. The remaining children go without siblings. How many families are going on the trip?

A) 19 **B)** 25 **C)** 31 **D)** 36 **E)** 48



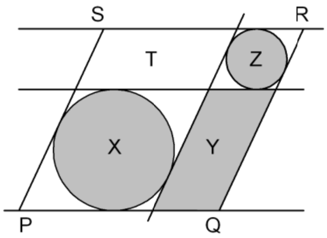
14) How many graphs of the functions $y = x^2$, $y = -x^2$, $y = +\sqrt{x}$, $y = -\sqrt{x}$, $y = +\sqrt{-x}$, $y = -\sqrt{-x}$, $y = +\sqrt{|x|}$, $y = -\sqrt{|x|}$ are included in the sketch on the right?

A) none **B)** 2 **C)** 4 **D)** 6 **E)** all 8



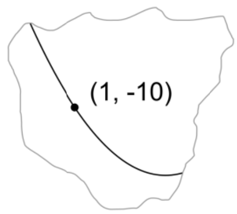
15) The rear window wiper of a car is made in a way so that the rod r and the wiper blade w are equally long and are connected at an angle α. The wiper rotates around the centre of rotation O and wipes over the area shown on the right. Calculate the angle β between the right edge of the cleaned area and the tangent of the curved upper edge.

A) $\frac{3\pi-\alpha}{2}$ **B)** $\pi - \frac{\alpha}{2}$ **C)** $\frac{3\pi}{2} - \alpha$ **D)** $\frac{\pi}{2} + \alpha$ **E)** $\pi + \frac{\alpha}{2}$



16) We have three horizontal lines and three parallel, sloped lines. Both of the circles shown touch four of the lines. X, Y and Z are the areas of the grey regions. D is the area of the parallelogram PQRS. At least how many of the areas X, Y, Z and D does one have to know in order to be able to determine the area of the parallelogram T?

A) 1 **B)** 2 **C)** 3 **D)** 4
E) T cannot be determined from X, Y, Z and D.



17) In the (x,y)-plane the co-ordinate axes are positioned as usual. Point A(1, -10) which is on the parabola $y = ax^2 + bx + c$ was marked. Afterwards the co-ordinate axis and the majority of the parabola were deleted. Which of the following statements could be false?

A) $a > 0$ **B)** $b < 0$ **C)** $a + b + c < 0$ **D)** $b^2 > 4ac$ **E)** $c < 0$

18) The sides AB, BC, CD, DE, EF and FA of a hexagon all touch the same circle. The measurements of the sides AB, BC, CD, DE and EF are in this order 4, 5, 6, 7 and 8. How long is side FA?

A) 9 **B)** 8 **C)** 7 **D)** 6

E) The length cannot be determined with this information.

19) Which is the smallest possible positive, whole number value of the expression

$\frac{K \cdot A \cdot N \cdot G \cdot A \cdot R \cdot O \cdot O}{G \cdot A \cdot M \cdot E}$ if different letters stand for different digits not equal to 0 and the same letters stand for the same digits?

A) 1 **B)** 2 **C)** 3 **D)** 5 **E)** 7

20) The brothers Gerhard and Günther pass on information about the members of their chess club. Gerhard says: "All members of our club are male with five exceptions." Günther says: "In each group of six members there are at least four female members." How many members does the chess club have?

A) 6 **B)** 7 **C)** 8 **D)** 12 **E)** 18

- 5 Point Questions -

21) In a drum there are a number of balls. A different positive whole number is written on each ball. On 30 of the balls numbers that are divisible by 6 are written, on 20 balls numbers that are divisible by 7 are written and on 10 balls numbers that are divisible by 42 are written. What is the minimum number of balls in the drum?

A) 30 **B)** 40 **C)** 53 **D)** 54 **E)** 60

22) Given are the two arithmetic sequences 5, 20, 35, … and 35, 61, 87,… . How many different arithmetic sequences of positive whole numbers do both sequences have as subsequences?

A) 1 **B)** 3 **C)** 5 **D)** 26 **E)** infinite

23) The function sequence $f_1(x), f_2(x), \cdots,$ fulfills the conditions $f_1(x) = x$ and $f_{n+1}(x) = \frac{1}{1-f_{n(x)}}$

Determine the value of $f_{2011}(2011)$.

A) 2011 **B)** $-\frac{1}{2010}$ **C)** $\frac{2010}{2011}$ **D)** 1 **E)** -2011

24) In a box there are red and green balls. If two balls are taken out of the box at random, the probability of them both being the same colour is ½. Which of the following could be the total number of balls in the box?

A) 81 **B)** 101 **C)** 1000 **D)** 2011 **E)** 10001

25) An airline does not charge for luggage if it is below a certain weight. For each additional kg of weight there is a charge. Mr. and Mrs. Raiss had 60 kg of luggage and paid 3 €. Mr. Wander also had 60 kg of luggage but had to pay 10.50 €. How many kg of luggage per passenger were transported for free?

A) 10 **B)** 18 **C)** 20 **D)** 25 **E)** 39

26) Determine the sum of all positive whole numbers x less than 100 so that $x^2 - 81$ is a multiple of 100.

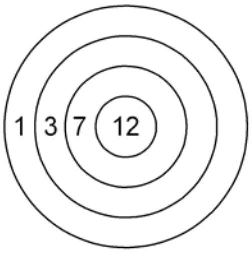
A) 200 **B)** 100 **C)** 90 **D)** 81 **E)** 50

27) An archer tries his art on the target shown below on the right. With each of his three arrows he always hits the target. How many different scores could he total with three arrows?

A) 13 **B)** 17 **C)** 19 **D)** 20 **E)** 21

28) Let a, b and c be positive whole numbers for which the following holds true $a^2 = 2b^3 = 3c^5$. What is the minimum number of factors of abc if 1 and abc are counted as well?

A) 30 **B)** 49 **C)** 60 **D)** 77 **E)** 1596



29) Twenty different positive whole numbers are written into a 4×5 table. Two numbers in cells that have one common sideline, always have a common factor greater than 1. Determine the smallest possible value of n, if n is to be the biggest number in the table.

A) 21 **B)** 24 **C)** 26 **D)** 27 **E)** 40

30) A 3 × 3 × 3 die is assembled out of 27 identical small dice. One plane, perpendicular to one of the space diagonals of the die goes through the midpoint of the die. How many of the smaller dice are cut by this plane?

A) 17 **B)** 18 **C)** 19 **D)** 20 **E)** 21