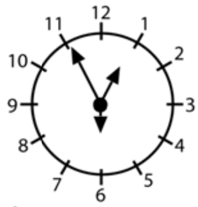
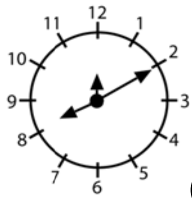
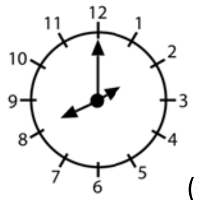
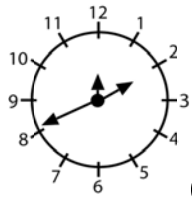
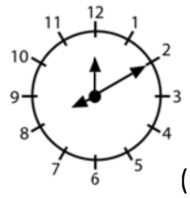
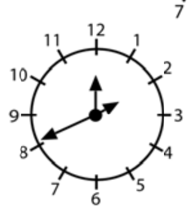
Mathematics Kangaroo 2012
Group Student (Grade 11 onwards)
Austria - 15.3.2012

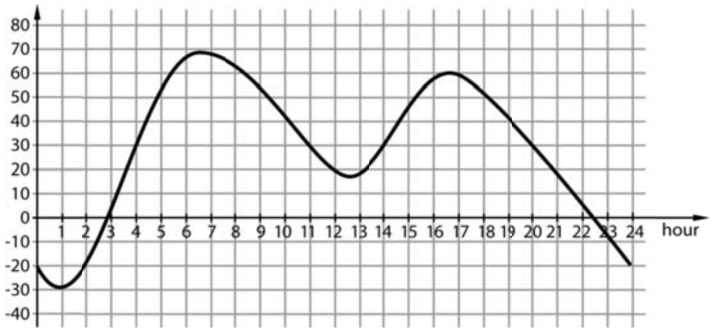
- 3 Point Questions -



1. A clock has three hands in different lengths (for seconds, minutes and hours). We don't know the length of each hand but we know that the clock shows the correct time. At 12:55:30 the hands are in the positions shown on the right. What does the clockface look like at 8:10:00?

(A)  (B)  (C)  (D)  (E) 

2. The water level in a port rises and falls on a certain day as shown in the diagram. How many hours on that day was the water level over 30 cm?



(A) 5 (B) 6 (C) 7 (D) 9 (E) 13

3. How many different rectangles with area 60 and whole numbered side lengths are there?

(A) 8 (B) 6 (C) 5 (D) 4 (E) 3

4. The positive whole numbers are being coloured in order, in red, blue and green, i.e. 1 red, 2 blue, 3 green, 4 red, 5 blue, 6 green, and so on. Which colour could the sum of a red number and a blue number be?

(A) any colour (B) red or blue (C) green only (D) red only (E) blue only

5. The number $\sqrt[3]{2\sqrt{2}}$ is equal to

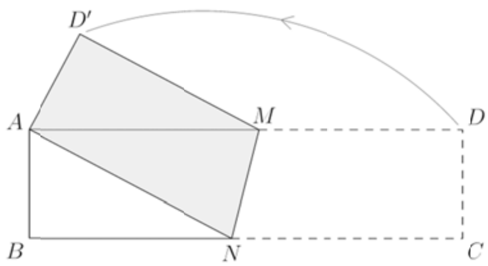
(A) 1 (B) $\sqrt{2}$ (C) $\sqrt[6]{4}$ (D) $\sqrt[3]{4}$ (E) 2

6. In a list of five numbers the first number is 2 and the last one is 12. The product of the first three numbers is 30, of the middle three 90 and of the last three 360. What is the middle number in that list?

2				12

(A) 3 (B) 4 (C) 5 (D) 6 (E) 10

7. A rectangular piece of paper ABCD with the measurements 4 cm x 16 cm is folded along the line MN so that point C coincides with point A as shown. How big is the area of the quadrilateral ANMD'?



(A) 28 cm^2 (B) 30 cm^2 (C) 32 cm^2 (D) 48 cm^2 (E) 56 cm^2

8. The sum of the digits of a nine digit number is 8. How big is the product of the digits of this number?

(A) 0 (B) 1 (C) 8 (D) 9 (E) 9!

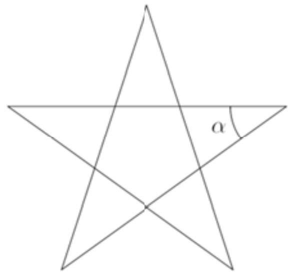
9. The biggest possible natural number n, for which $n^{200} < 5^{300}$ holds true is

(A) 5 (B) 6 (C) 8 (D) 11 (E) 12

10. The age of Quintus is a two digit power of five and the age of Sekundus is a two digit power of two. If one adds the digits of their ages the total obtained is an odd number. How big is the product of the digits of their ages?

(A) 240 (B) 2012 (C) 60 (D) 50 (E) 300

- 4 Point Questions -



11. How big is the angle α in the regular five-sided star shown?

(A) 24° (B) 30° (C) 36° (D) 45° (E) 72°

12. A real number x fulfills the condition $x^3 < 64 < x^2$. Which of the following statements is definitely true?

(A) $0 < x < 64$ (B) $-8 < x < 4$ (C) $x > 8$ (D) $-4 < x < 8$ (E) $x < -8$

13. A travel agency organises four different trips for a certain group. Each trip has a participation rate of 80%. What is the minimum percentage of the group which has taken part in all four roundtrips?

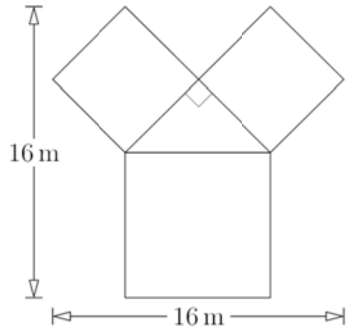
(A) 80 % (B) 60 % (C) 40 % (D) 20 % (E) 16 %

14. For a ski race consecutive starting numbers are handed out. One number was accidentally given out twice. The sum of all the numbers handed out is 857. Which number was given out twice?

(A) 4 (B) 16 (C) 25 (D) 37 (E) 42

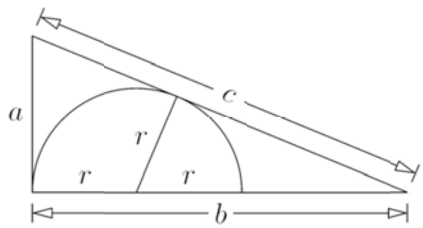
15. In one class a test did not yield a very successful result because the average mark was exactly 4. The boys have done slightly better with an average mark of 3.6, while the girls have received an average mark of 4.2. Which of the following statements is correct?

(A) There are twice as many boys as girls.
(B) There are 4 times as many boys as girls.
(C) There are twice as many girls as boys.
(D) There are 4 times as many girls as boys.
(E) There are equally many boys and girls.



16. In the diagram we see a rose bed. White roses are growing in the squares that are equally big, red ones are in the big square and yellow ones in the right-angled triangle. The bed has width and height 16 m. How big is the area of the bed?

(A) 114 m^2 (B) 130 m^2 (C) 144 m^2 (D) 160 m^2 (E) 186 m^2

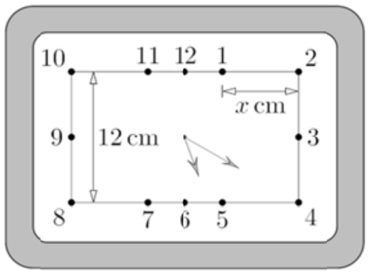


17. A right-angled triangle with side lengths a = 8, b = 15 and c = 17 is given. How big is the radius r of the inscribed semicircle shown?

(A) 2·4 (B) 3 (C) 3·75 (D) 4·8 (E) 6

18. A square ABCD has side-length 2. E is the midpoint of AB and F the midpoint of AD. G is a point on the line CF with 3CG = 2GF. How big is the area of the triangle BEG?

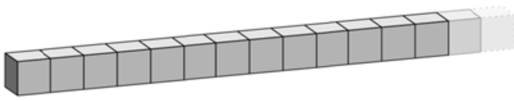
(A) $\frac{7}{10}$ (B) $\frac{4}{5}$ (C) $\frac{8}{5}$ (D) $\frac{3}{5}$ (E) $\frac{6}{5}$



19. The clock shown has a rectangular clock face, the hands however move as usual in a constant circular pattern. How big is the distance x of the digits 1 and 2 (in cm), if the distance between the numbers 8 and 10 is given as 12 cm?

(A) $3\sqrt{3}$ (B) $2\sqrt{3}$ (C) $4\sqrt{3}$ (D) $2+\sqrt{3}$ (E) $12-3\sqrt{3}$

20. Renate wants to glue together a number of ordinary dice (whose number of points on opposite sides always adds up to 7) to form a “dicebar” as shown. Doing this she only wants to glue sides together with an equal number of points. She wants to make sure that the sum of all points on the non-glued sides equals 2012. How many dice does she have to glue together?



(A) 70 (B) 71 (C) 142 (D) 143
(E) It is impossible to obtain exactly 2012 points on the non-glued together sides.

- 5 Point Questions -

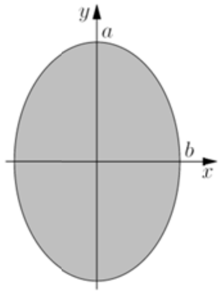
21. Which of the following functions fulfills for all x $\neq$ 0 the condition $f\left(\frac{1}{x}\right) = f(x)$?

(A) $f(x) = \frac{2}{x}$ (B) $f(x) = \frac{1}{x+1}$ (C) $f(x) = 1 + \frac{1}{x}$ (D) $f(x) = \frac{1}{x}$ (E) $f(x) = x + \frac{1}{x}$

22. The solution set of the inequality |x| + |x-3| > 3 is

(A) $]-\infty,0[\ \cup]3,+\infty[$ (B) $]-3, 3[$ (C) $]-\infty,-3[$

(D) $]-3,+\infty[$ (E) **R**



23. Let a > b. If the ellipse shown rotates about the x-axis an ellipsoid E_x with volume $Vol(E_x)$ is obtained. If it rotates about the y-axis an ellipsoid E_y with volume $Vol(E_y)$ is obtained. Which of the following statements is true?

(A) $E_x = E_y$ and $Vol(E_x) = Vol(E_y)$ (B) $E_x = E_y$ but $Vol(E_x) \neq Vol(E_y)$

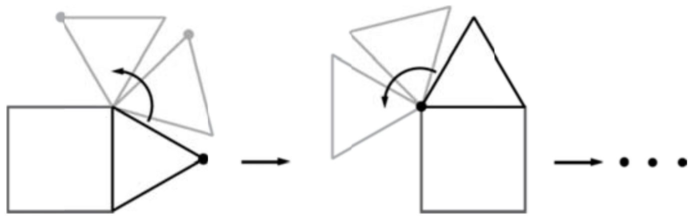
(C) $E_x \neq E_y$ and $Vol(E_x) > Vol(E_y)$ (D) $E_x \neq E_y$ and $Vol(E_x) < Vol(E_y)$

(E) $E_x \neq E_y$ but $Vol(E_x) = Vol(E_y)$

24. In a game with fractions I am allowed to carry out two operations, namely either increase the numerator by 8 or increase the denominator by 7 without simplifying during the game. Starting with the fraction $\frac{7}{8}$ after n such operations I again obtain a fraction with equal value. What is the smallest value of n?

(A) 56 (B) 81 (C) 109 (D) 113 (E) This value cannot be obtained.

25. An equilateral triangle is being rolled around a unit square as shown. How long is the path that the point shown covers, if the point and the triangle are both back at the start for the first time?



(A) 4π (B) $\frac{28}{3}$π (C) 8 π (D) $\frac{14}{3}$ π (E) $\frac{21}{2}$ π

26. How many permutations (x_1, x_2, x_3, x_4) of the set {1, 2, 3, 4} have property that the number $x_1 x_2 + x_2 x_3 + x_3 x_4 + x_4 x_1$ is divisible by 3?

(A) 8 (B) 12 (C) 14 (D) 16 (E) 24

27. After an especially intense lesson the graph of the function $y = x^2$ was still on the board as well as 2012 straight lines parallel to the straight line with the equation y = x, which each intersected the parabola in two points. How big is the sum of all x-coordinates of the intersections of the straight lines with the parabola?

(A) 0 (B) 1 (C) 1006 (D) 2012 (E) The number depends on the choice of the straight lines.

28. Three corners of a die (not all on one face) have the coordinates P(3,4,1), Q(5,2,9) and R(1,6,5). What are the coordinates of the midpoint of the die?

(A) A(4,3,5) (B) B(2,5,3) (C) C(3,4,7) (D) D(3,4,5) (E) E(2,3,5)

29. In the sequence 1, 1, 0, 1, -1, … the first two terms a_1 and a_2 are each 1. The third term is the difference of the previous two and $a_3 = a_1 - a_2$ holds true. The fourth one is the sum of the previous two with $a_4 = a_2 + a_3$. Then $a_5 = a_3 - a_4$, $a_6 = a_4 + a_5$, and so on, alternating difference and sum. How big is the sum of the first 100 terms of this sequence?

(A) 0 (B) 3 (C) -21 (D) 100 (E) -1

30. Gerhard chooses two numbers a and b from the set {1, 2, 3, …, 26}. The product a×b of these two numbers is equal to the sum of the remaining 24 numbers from this set. How big is |a-b|?

(A) 10 (B) 9 (C) 7 (D) 2 (E) 6