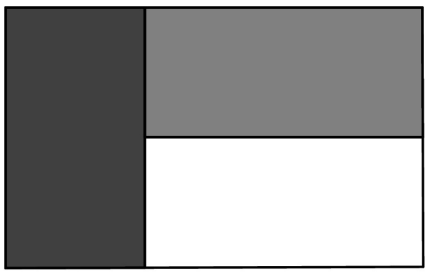
Känguru der Mathematik 2019
Level Student (Schulstufe 11, 12 and 13)
Austria – 21. 3. 2019

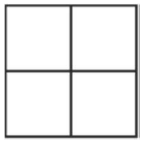
- 3 Point Examples -

1. The flag of Kangoraland is a rectangle which is split into three equal rectangles as shown.
How big is the ratio of the side lengths of the white rectangle?



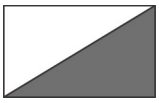
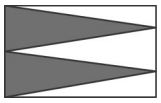
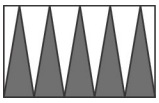
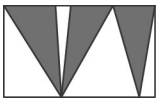
(A) $1:2$ (B) $2:3$ (C) $2:5$ (D) $3:7$ (E) $4:9$

2. The numbers 1, 2, 3 and 4 are inserted into different cells of the 2×2 table shown. Then the sums of the numbers in each row and column are determined. Two of these sums are 4 and 5. How big are the two remaining sums?

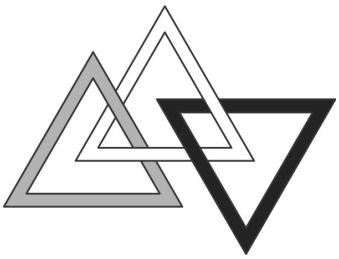


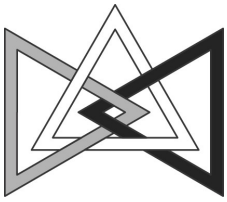
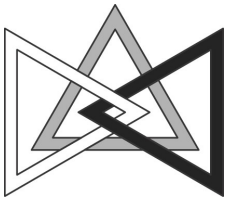
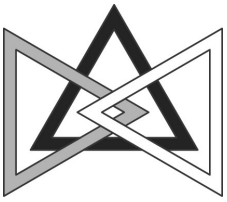
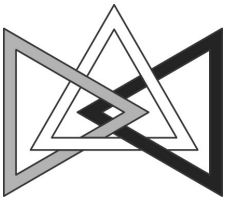
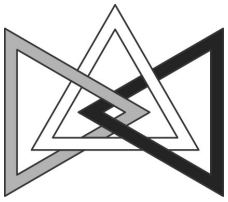
(A) 6 and 6 (B) 3 and 5 (C) 4 and 5 (D) 4 and 6 (E) 5 and 6

3. A rectangle is coloured in five different ways as shown. In which picture is the grey area biggest?

(A)  (B)  (C)  (D)  (E) 

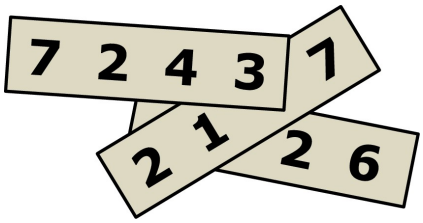
4. Three triangles are connected to each other as shown. In which of the following pictures are the three triangles connected in the same way?



(A)  (B)  (C)  (D)  (E) 

5. A pyramid has 23 triangular faces. How many edges does this pyramid have?
(A) 23 (B) 24 (C) 46 (D) 48 (E) 69

6. Three four-digit numbers are written onto three separate pieces of paper as shown. The sum of the three numbers is 11126. Three of the digits in the picture are hidden. Which are the three hidden digits?

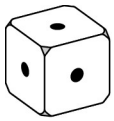


(A) 1, 4 and 7 (B) 1, 5 and 7 (C) 3, 3 and 3 (D) 4, 5 and 6 (E) 4, 5 and 7

7. Reading from the left, what is the first digit of the smallest positive integer whose digit sum is 2019?
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

8. How many of the numbers from 2^{10} to 2^{13} (including these two numbers) are divisible by 2^{10}?
(A) 2 (B) 4 (C) 6 (D) 8 (E) 16

9. Each side of a die is marked with either 1, 2 or 3 dots so that the probability of rolling a 1 is equal to $\frac{1}{2}$, the probability of rolling a 2 is equal to $\frac{1}{3}$ and the probability of rolling a 3 is equal to $\frac{1}{6}$.
Which of these pictures cannot be a picture of this particular die?

(A)  (B)  (C)  (D)  (E) 

10. Every day the three kangaroos Alex, Bob and Carl go for a walk. If Alex does not wear a hat, Bob wears a hat. If Bob does not wear a hat, Carl wears a hat. Today Carl does not wear a hat. Which kangaroos can one say for sure are wearing a hat today?

(A) only Alex and Bob (B) only Alex (C) Alex, Bob and Carl

(D) neither Alex nor Bob (E) only Bob

- 4 Point Examples -

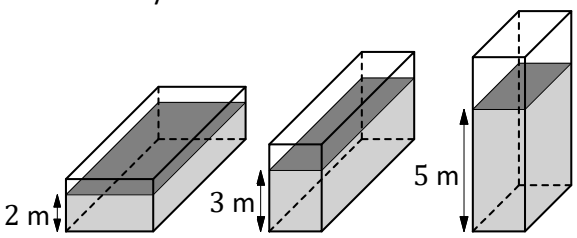
11. Which is the highest power of three that divides the number $7! + 8! + 9!$?
(Hint: The expression n! is defined by n! = n·(n-1)·(n-2)·…·3·2·1.)

(A) 3^2 (B) 3^4 (C) 3^5 (D) 3^6 (E) a power of three greater than 3^6

12. In this school year the number of boys in my class has increased by 20% compared to the previous year and the number of girls has decreased by 20%. There is now one person more than before in this class.
Which of the following numbers could be the current number of students in my class?

(A) 22 (B) 26 (C) 29 (D) 31 (E) 34

13. A cuboid-shaped container that is not filled completely contains 120 m³ of water. The depth of the water is either 2 m or 3 m or 5 m, depending on which side the container is actually standing on (drawings not to scale).
How big is the volume of the container?

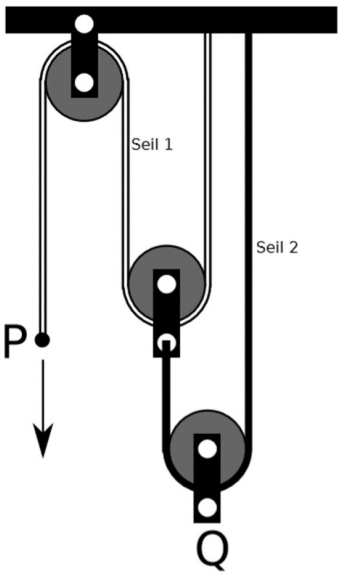


(A) 160 m³ (B) 180 m³ (C) 200 m³ (D) 220 m³ (E) 240 m³

14. Michael invents a new operation „$\diamond$“ for real numbers that is defined by $x \diamond y = y - x$. Which of the following statements is definitely true if the numbers a, b, and c fulfill the condition $(a \diamond b) \diamond c = a \diamond (b \diamond c)$?

(A) $a = b$ (B) $b = c$ (C) $a = c$ (D) $a = 0$ (E) $c = 0$

15. The system shown consists of three pulleys that are connected to each other via two ropes. P, the end of one rope, is pulled down by 24 cm.
By how many centimeters does point Q move upwards?



(A) 24 (B) 12 (C) 8 (D) 6 (E) $\frac{24}{5}$

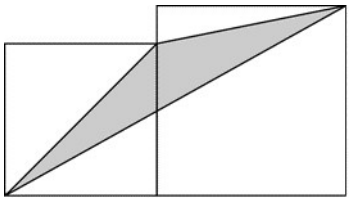
16. A positive integer n is called good, if it's biggest factor (apart from n itself) is equal to $n - 6$. How many good positive integers are there?

(A) 1 (B) 2 (C) 3 (D) 6 (E) infinitely many

17. There are five balls in a box. Four of which contain chocolate, one contains one boiled sweet. Johann and Maria take it in turns to draw a ball from the box without replacing it. Whoever draws the boiled sweet wins. Johann starts.
How big is the probability that Maria wins?

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\frac{1}{2}$ (D) $\frac{5}{6}$ (E) $\frac{1}{3}$

18. The diagram shows two adjoining squares with side lengths a and b (with $a < b$). How big is the area of the grey triangle?

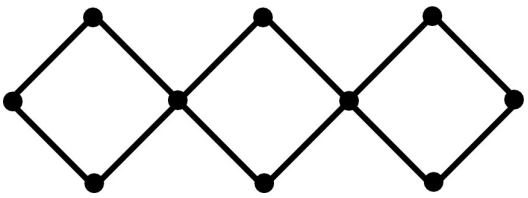


(A) $\sqrt{ab}$ (B) $\frac{1}{2}a^2$ (C) $\frac{1}{2}b^2$ (D) $\frac{1}{4}(a^2 + b^2)$ (E) $\frac{1}{2}(a^2 + b^2)$

19. What is the biggest integer smaller than $\sqrt{20 + \sqrt{20 + \sqrt{20 + \sqrt{20 + \sqrt{20}}}}}$?

(A) 4 (B) 5 (C) 6 (D) 20 (E) 25

20. The points of intersection of the network of bars shown are labelled with the numbers 1 to 10. The sums S of the four numbers on the vertices of each square are all the same.
What is the minimum value of S?



(A) 18 (B) 19 (C) 20 (D) 21 (E) 22

- 5 Point Examples -

21. Let a be the sum of all positive factors of 1024 and b be the product of all positive factors of 1024. (Hint: 1 and 1024 are also factors of 1024.) Then

(A) $(a-1)^5 = b$ (B) $(a+1)^5 = b$ (C) $a^5 = b$ (D) $a^5 - 1 = b$ (E) $a^5 + 1 = b$

22. Which is the set of all parameters a for which the equation $2 - |x| = ax$ has exactly two solutions?

(A) $]-\infty;-1]$ (B) $]-1;1[$ (C) $[1;+\infty[$ (D) $\{0\}$ (E) $\{-1;1\}$

23. In order to determine the result of the calculation $\frac{a+b}{c}$ (a, b and c are positive integers), Sara inserts into a calculator $a + b \div c =$ and obtains the result 11. Then she inserts $b + a \div c =$ and is surprised that the result is now 14. She realises that the calculator follows the rules for the order of operations and does division before addition.

What is the actual result of the calculation $\frac{a+b}{c}$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

24. Consider a cube. How many planes are there that go through at least three vertices of this cube?

(A) 6 (B) 8 (C) 12 (D) 16 (E) 20

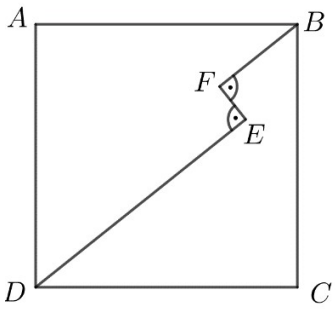
25. Four different straight lines go through the origin of the co-ordinate-system. They intersect the parabola $y = x^2 - 2$ at eight points. What could be the product of the x-co-ordinates of these eight points?

(A) only 16 (B) only -16 (C) only 8 (D) only -8 (E) There is more than one possible value.

26. For how many integers n is $|n^2 - 2n - 3|$ a prime number?

(A) 1 (B) 2 (C) 3 (D) 4 (E) infinitely many

27. A path $DEFB$ with $DE \perp EF$ and $EF \perp FB$ lies within the square $ABCD$ as shown. We know that $DE = 5$, $EF = 1$ and $FB = 2$.
What is the side length of the square?

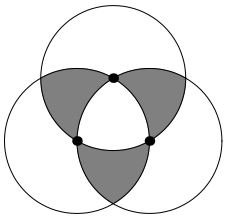


(A) $3\sqrt{2}$ (B) $\frac{7\sqrt{2}}{2}$ (C) $\frac{11}{2}$ (D) $5\sqrt{2}$ (E) another value

28. The sequence $a_1, a_2, a_3, \ldots$ starts with $a_1 = 49$. To work out a_{n+1} for $n \geq 1$ you add 1 to the digit sum of a_n and square the result. So e.g. $a_2 = (4+9+1)^2 = 196$. Work out a_{2019}.

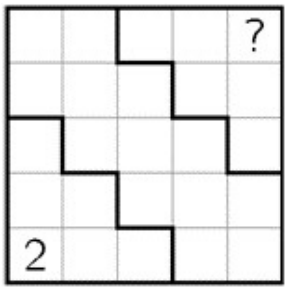
(A) 121 (B) 25 (C) 64 (D) 400 (E) 49

29. Three circles with radius 2 are drawn in such a way that each time one of the points of intersection of two circles is identical with the centre of the third circle.
How big is the area of the grey zone?



(A) π (B) 3π (C) $\frac{\pi}{2}$ (D) 2π (E) 4π

30. numbers are to be placed into the square grid shown, so that each of the numbers 1, 2, 3, 4 and 5 appears exactly once in each row and in each column. Furthermore sthe sum of all numbers in the three black-bordered sections should always be the same.
Which number has to be written into the top right cell?



(A) 1 (B) 2 (C) 3 (D) 4 (E) 5