

KÄNGURU DER MATHEMATIK 2023

16. 3. 2023

Level: Junior, Grade: Schulstufe 9 + 10



Full name:

School:

Class:

Time: 75 min.

30 starting points

each correct answer to questions 1. – 10.: 3 points

each correct answer to questions 11. – 20.: 4 points

each correct answer to questions 21. – 30.: 5 points

each questions left unanswered: 0 points

each incorrect answer: minus $\frac{1}{4}$ of the points for the question

Please write the letter (A, B, C, D, E) of the correct answer in the square under the question number (1 bis 30). Write clearly and carefully!

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|----|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| | | | | | | | | | |

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|----|----|----|----|----|----|----|----|----|----|
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| | | | | | | | | | |

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|----|----|----|----|----|----|----|----|----|----|
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| | | | | | | | | | |

Zustimmungserklärung zur Datenverarbeitung für den österreichischen Wettbewerb „Känguru der Mathematik“

Mit meiner Unterschrift gebe ich das Einverständnis, dass meine angeführten personenbezogenen Daten (Vor- und Zuname, Klasse, Schulstufe, Schulstandort und Schularart) zum Zweck der Organisation und Durchführung des Wettbewerbs, der Auswertung der Wettbewerbsergebnisse (Ermitteln der erreichten Punkte und Prozentzahlen), des Erstellens von schulweiten Reihungen, sowie zur Erstellung und Veröffentlichung der Siegerlisten auf unserer Vereinshomepage (sofern mindestens 50 % der zu erreichenden Punktezahl erlangt werden bzw. ich unter den besten 10 einer Kategorie liege) verwendet werden dürfen.

Betroffenenrechte

Die Verwendung dieser Daten ist bis 31. Dezember des 2. Folgejahres gestattet. Nach diesem 31. Dezember werden Vor- und Zuname, die Klasse und der Schulstandort gelöscht, wobei dieser durch die Angabe des Bundeslandes ersetzt wird. Die Verwendung der auf diese Art anonymisierten Daten ist nur mehr für statistische Zwecke auf der Grundlage der DSGVO erlaubt.

Ich habe ein Recht auf Auskunft über meine gespeicherten personenbezogenen Daten, sowie das Recht auf Berichtigung, Datenübertragung, Widerspruch, Einschränkung der Bearbeitung sowie Sperrung oder Löschung unrichtig verarbeiteter Daten.

Ich kann die erteilte Einwilligung jederzeit auf der Homepage des Vereines Känguru der Mathematik unter www.kaenguru.at mittels des dafür bereitgestellten Formulars mit Wirkung für die Zukunft widerrufen (Art. 21 Abs. 1 DSGVO).

Ein Widerruf hat zur Folge, dass die personenbezogenen Daten nach gegenseitiger Rücksprache innerhalb von 31 Tagen gelöscht werden.

Durch den Widerruf wird die Rechtmäßigkeit der aufgrund der Einwilligung bis zum Widerruf erfolgten Verarbeitung nicht berührt. (Art. 7 Abs. 2 DSGVO)

Ort, Datum

Unterschrift



Information über den Känguruwettbewerb: www.kaenguru.at
Wenn du mehr in dieser Richtung machen möchtest, gibt es die Österreichische Mathematikolympiade.
Infos unter: www.oemo.at

Känguru der Mathematik 2023

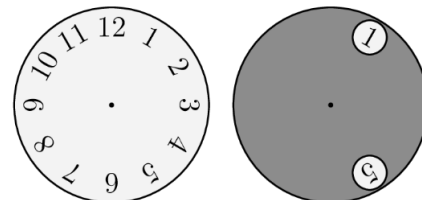
Level Junior (Schulstufe 9 and 10)

Austria – 16. 3. 2023



- 3 Point Examples -

1. A dark disc with two holes is placed on the dial of a watch as shown in the diagram. The dark disc is now rotated so that the number 10 can be seen through one of the two holes.

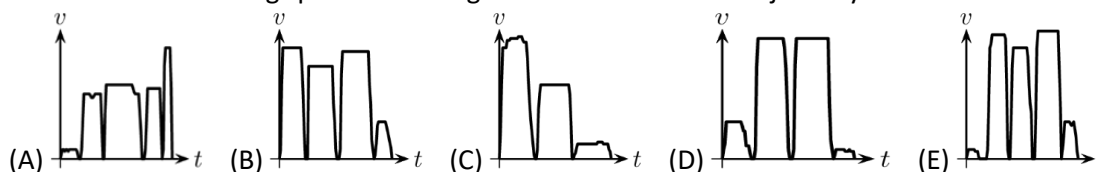


Which of the numbers could one see through the other hole now?

- (A) 2 and 6 (B) 3 and 7 (C) 3 and 6 (D) 1 and 9 (E) 2 and 7

2. On her way to school Maria first had to run to the underground, she exited from that after two stops and subsequently walked the rest of the way by foot all the way to school.

Which of the following speed-time-diagrams best describes her journey to school?

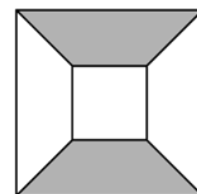


3. The two integers m and n are positive and odd. Which of the following numbers is odd?

- (A) $m \cdot n + 2$ (B) $(m + 1) \cdot (n + 1)$ (C) $m + n + 2$ (D) $m \cdot (n + 1)$ (E) $m + n$

4. A small square with side length 4 cm is drawn within a big square with side length 10 cm; their sides are parallel to each other (see diagram). What percentage of the figure is shaded?

- (A) 25 % (B) 30 % (C) 40 % (D) 42 % (E) 45 %

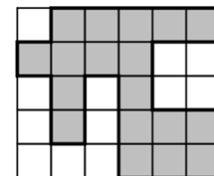


5. Today is Thursday. What day of the week is it in 2023 days?

- (A) Tuesday (B) Wednesday (C) Thursday (D) Friday (E) Saturday

6. The big rectangle shown is divided into 30 equally big squares. The perimeter of the area shaded in grey is 240 cm. How big is the area of the big rectangle?

- (A) 480 cm² (B) 750 cm² (C) 1080 cm² (D) 1920 cm² (E) 2430 cm²

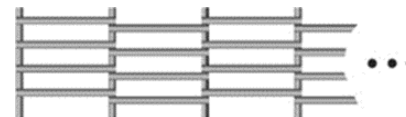


7. If one adds the ages of all members of a family of five together, one gets 80. The two youngest children are 6 and 8 years old. How big was the sum of the ages of the family members 7 years ago?

- (A) 35 (B) 36 (C) 44 (D) 46 (E) 66

8. A straight wooden fence is made up of vertical beams stuck in the ground which are each connected to the next beam by 4 horizontal beams. The fence begins and ends with a vertical beam. Out of how many beams could such a fence be made?

- (A) 95 (B) 96 (C) 97 (D) 98 (E) 99



9. How many pairs of positive integers (a, b) fulfil the equation $\frac{a}{5} = \frac{7}{b}$?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

10. After playing 200 games of chess, Beth's winning rate is exactly 49 %.

What is the minimum number of games she has to still play to increase her winning rate to 50 %?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

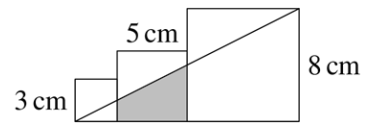
- 4 Points Examples -

11. Jennifer wants to save water. She reduces the water pressure and thus reduces the water usage by one quarter. Furthermore, she reduces the time she takes a shower by one quarter.

By which fraction in total does she reduce the water usage for her shower?

- (A) by $\frac{1}{4}$ (B) by $\frac{3}{8}$ (C) by $\frac{1}{16}$ (D) by $\frac{5}{12}$ (E) by $\frac{7}{16}$

12. The diagram shows three adjacent squares with side lengths 3 cm, 5 cm and 8 cm. How big is the area of the shaded in trapezium?

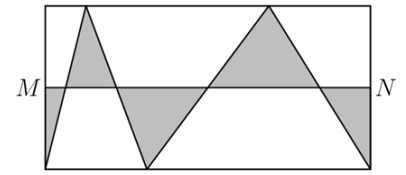


- (A) 13 cm^2 (B) $\frac{55}{4} \text{ cm}^2$ (C) $\frac{61}{4} \text{ cm}^2$ (D) $\frac{65}{4} \text{ cm}^2$ (E) $\frac{69}{4} \text{ cm}^2$

13. A rope with length 95 m is cut into three pieces so that each piece is half as long again as the respective previous piece. How long is the longest of the three pieces?

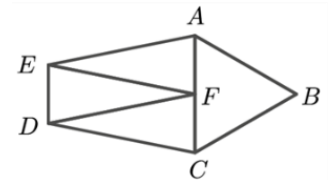
- (A) 39 m (B) 42 m (C) 45 m (D) 48 m (E) 54 m

14. The points M and N are the midpoints of two sides of the big rectangle (see diagram). Which part of the area of the big rectangle is shaded?



- (A) $\frac{1}{6}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{1}{3}$ (E) $\frac{1}{2}$

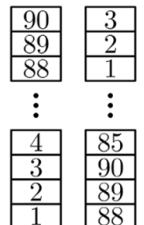
15. The pentagon $ABCDE$ is split into four triangles that all have the same perimeter (see diagram). Triangle ABC is equilateral and the triangles AEF , DFE and CDF are congruent isosceles triangles.



How big is the ratio of the perimeter of the pentagon $ABCDE$ to the perimeter of the triangle ABC ?

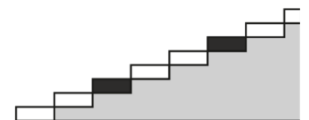
- (A) 2 (B) $\frac{3}{2}$ (C) $\frac{4}{3}$ (D) $\frac{5}{3}$ (E) $\frac{5}{2}$

16. A tower consists of blocks that are labelled from bottom to top with the numbers from 1 to 90. Bob uses these blocks to build a new tower. For each step he takes the top three blocks from the old tower and places them on the new tower without changing their order (see diagram). How many blocks are there in the new tower between the blocks with the numbers 39 and 40?



- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

17. A staircase has 2023 steps. Every third step is coloured in black. The first seven steps of this staircase can be fully seen in the diagram. Anita walks up the staircase and steps on each step exactly once. She can start with either the right or the left foot and then steps down alternately with the right or left foot.



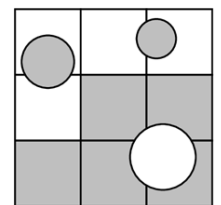
What is the minimum number of black steps she sets her right foot on?

- (A) 332 (B) 333 (C) 336 (D) 337 (E) 672

18. We call a positive integer *powerfree* if none of its digits can be written as a power of an integer with an exponent bigger than 1. For example, the number 53 is powerfree, but the number 54 is not powerfree since $4 = 2^2$. Which one of the following numbers is the difference between the biggest and the smallest two-digit powerfree numbers?

- (A) 24 (B) 55 (C) 63 (D) 88 (E) 89

19. A square with side length 30 cm is split into 9 squares. The big square contains three circles with radii 5 cm (bottom right), 4 cm (top left) as well as 3 cm (top right) as seen in the diagram. How many cm^2 are shaded in grey?



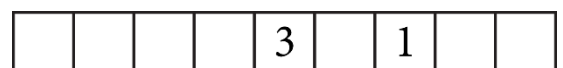
- (A) $500 + 25\pi$ (B) 500 (C) $400 + 50\pi$ (D) 400 (E) $500 - 25\pi$

20. The arithmetic mean of five different prime numbers is an integer number. What is the smallest possible number of this arithmetic mean?

- (A) 5 (B) 6 (C) 7 (D) 8 (E) 9

- 5 Points Examples -

21. The numbers from 1 to 9 should be distributed among the 9 squares in the diagram according to the following rules:



There should be one number in each square. The sum of three adjacent numbers is always a multiple of 3. The numbers 3 and 1 are already placed.

How many ways are there to place the remaining numbers?

- (A) 9 (B) 12 (C) 15 (D) 18 (E) 24

22. How many different ways are there to read the word *BANANA* in the following table if we can only cross to a field that shares an edge with the current field and we can use fields several times?

| | | |
|---|---|---|
| B | A | B |
| A | N | A |
| B | A | B |

- (A) 56 (B) 64 (C) 84 (D) 112 (E) 128

23. Starting with the four numbers

2, 0, 2, 3

the kangaroo-machine creates numbers according to the following rule: the next number is always the smallest non-negative integer that is different to the four directly previous numbers.

Which number is in position 2023?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

24. A circle with midpoint (75|30) and radius 10 is cut from a rectangle with vertices (0|0), (100|0), (100|50) and (0|50).

What is the gradient of the straight line that goes through the point (75|30) and divides the remaining part of the rectangle into two parts with equal area?

- (A) $\frac{1}{5}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{2}{5}$ (E) $\frac{2}{3}$

25. When Matilda's smartphone is fully charged it has a battery life of

32 hours if she phones continuously,

20 hours if she surfs the internet continuously and

80 hours if she does not use it at all.

Matilda boards a train with a half full battery. During her time on board she spends the same amount of time each on phoning, surfing the internet and not using the phone at all. Just when she arrives at her destination the battery is empty.

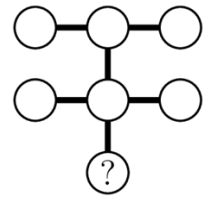
How many hours did the train ride take?

- (A) 10 (B) 12 (C) 14 (D) 16 (E) 18

26. Seven pairwise different single-digit numbers are distributed among the circles shown so that the product of the three numbers that are connected by a straight line is the same in all three cases.

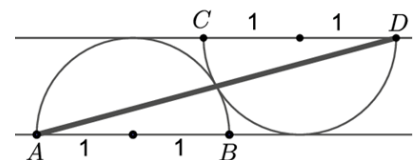
Which number is written in the circle with the question mark?

- (A) 2 (B) 3 (C) 4 (D) 6 (E) 8



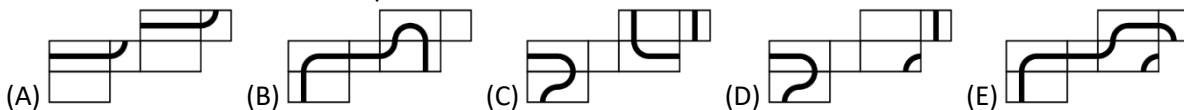
27. Consider the two touching semicircles with radius 1 and their diameters *AB* and *CD* respectively that are parallel to each other. The extensions of the two diameters are also tangents to the respective other semicircle (see diagram). How big is the square of the length *AD*?

- (A) 16 (B) $8+4\sqrt{3}$ (C) 12 (D) 9 (E) $5+2\sqrt{3}$



28. Leon has drawn a closed loop on the surface of a cuboid.

Which net **cannot** show his loop?



29. Several points are marked on a straight line. Renate marks another point between each pair of adjacent points. She repeats this process three more times. Now there are 225 points marked on this straight line.

How many points were marked to start with?

- (A) 15 (B) 20 (C) 25 (D) 29 (E) 32

30. The diagram shows the map of a big park. The park is split into several sections and the number in each section states its perimeter in km.

How big is the perimeter of the entire park in km?

- (A) 18 (B) 22 (C) 26 (D) 32 (E) 42

