KÄNGURU DER MATHEMATIK 2024
21. 3. 2024

Level: Junior, Grades 9 + 10

Full name:	
School:	
Class:	

Time: 75 min.
30 starting points
each correct answer to questions 1. – 10.: 3 points
each correct answer to questions 11. – 20.: 4 points
each correct answer to questions 21. – 30.: 5 points
each questions left unanswered: 0 points
each incorrect answer: minus ¼ of the points for the question

Please write the letter (A, B, C, D, E) of the correct answer in the square under the question number (1 bis 30). Write clearly and carefully!

1	2	3	4	5	6	7	8	9	10

11	12	13	14	15	16	17	18	19	20

21	22	23	24	25	26	27	28	29	30

Zustimmungserklärung zur Datenverarbeitung für den österreichischen Wettbewerb „Känguru der Mathematik“

☐Ich stimme zu, dass meine angeführten personenbezogenen Daten (Vor- und Zuname, Klasse, Schulstufe, Schulstandort und Schulart) zum Zweck der Organisation und Durchführung des Wettbewerbs, der Auswertung der Wettbewerbsergebnisse (Ermitteln der erreichten Punkte und Prozentzahlen), des Erstellens von schulweiten Reihungen, sowie zur Erstellung und Veröffentlichung der Siegerlisten auf unserer Vereinshomepage (sofern mindestens 50 % der zu erreichenden Punktezahl erlangt werden bzw. ich unter den besten 10 einer Kategorie liege) verwendet werden dürfen.

Betroffenenrechte
Die Verwendung dieser Daten ist bis 31. Dezember des 2. Folgejahres gestattet. Nach diesem 31. Dezember werden Vor- und Zuname, die Klasse und der Schulstandort gelöscht, wobei dieser durch die Angabe des Bundeslandes ersetzt wird. Die Verwendung der auf diese Art anonymisierten Daten ist nur mehr für statistische Zwecke auf der Grundlage der DSGVO erlaubt.

Ich habe ein Recht auf Auskunft über meine gespeicherten personenbezogenen Daten, sowie das Recht auf Berichtigung, Datenübertragung, Widerspruch, Einschränkung der Bearbeitung sowie Sperrung oder Löschung unrichtig verarbeiteter Daten.

Ich kann die erteilte Einwilligung jederzeit auf der Homepage des Vereines Känguru der Mathematik unter www.kaenguru.at mittels des dafür bereitgestellten Formulars mit Wirkung für die Zukunft widerrufen (Art. 21 Abs. 1 DSGVO).
Ein Widerruf hat zur Folge, dass die personenbezogenen Daten nach gegenseitiger Rücksprache innerhalb von 31 Tagen gelöscht werden.
Durch den Widerruf wird die Rechtmäßigkeit der aufgrund der Einwilligung bis zum Widerruf erfolgten Verarbeitung nicht berührt. (Art. 7 Abs. 2 DSGVO)

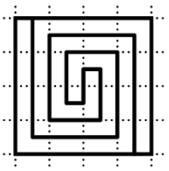
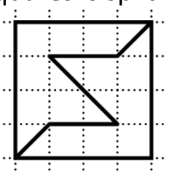
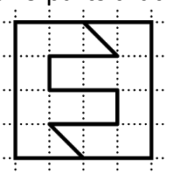
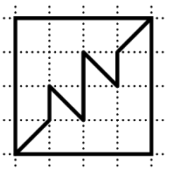
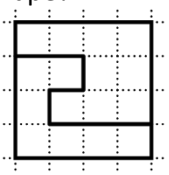
Ort, Datum Unterschrift

Information über den Känguruwettbewerb: www.kaenguru.at
Wenn du mehr in dieser Richtung machen möchtest,
gibt es die Österreichische Mathematikolympiade.
Infos unter: www.oemo.at

Känguru der Mathematik 2024
Level Junior (Schulstufe 9 and 10)
Austria – 21. 3. 2024

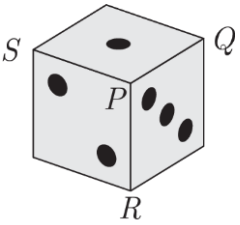
- 3 Point Examples -

1. Which of the shown squares is split into two parts that do **not** have the same shape?

(A)  (B)  (C)  (D)  (E) 

2. $\frac{2\times0.24}{20\times2.4}=?$ (A) 0.01 (B) 0.1 (C) 1 (D) 10 (E) 100

3. The number of points on opposite faces of a die is always 7. We define the *vertex sum* in a vertex, as the sum of the points on the faces that meet in the vertex. (E.g. the faces of the die with 1, 2 and 3 points meet in P, therefore, the vertex sum in point P is defined as 1+2+3 = 6.) Which of the following numbers is the biggest vertex sum in the vertices Q, R and S?

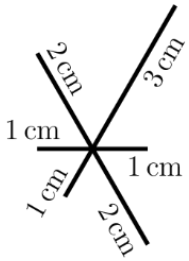


(A) 7 (B) 9 (C) 10 (D) 11 (E) 15

4. A type of hopscotch is played in the following way: each player jumps from one square to the next with the left foot, both feet, right foot, both feet, etc. alternately on the floor as shown. Maya plays this game and jumps into exactly 48 squares starting with the left foot. How often is her left foot on the floor in this game?

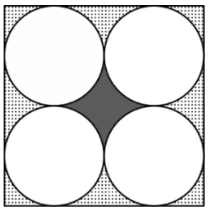
(A) 12 (B) 24 (C) 36 (D) 40 (E) 48

5. Tim wants to draw the figure shown without lifting his pencil off the paper.
He has to pass over some parts more than once. The segment lengths are stated in the figure. What is the minimum length of the total line he will draw if he can choose his starting point freely?



(A) 14 cm (B) 15 cm (C) 16 cm (D) 17 cm (E) 18 cm

6. The diagram shows a square with four touching circles of equal size. What is the ratio of the area of the black part to the grey part?

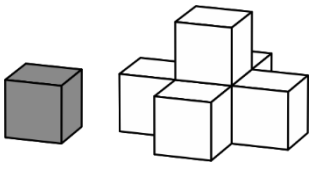


(A) $1:4$ (B) $1:3$ (C) $2:3$ (D) $3:4$ (E) $1:\pi$

7. Let *a* and *b* be numbers from the set {1,2,3,4,5,6}. For each pair (*a,b*) we draw a straight line with the equation *y = ax + b* and consider the triangle that this straight line forms with the co-ordinate axes. How many pairs (*a,b*) create an isosceles triangle?

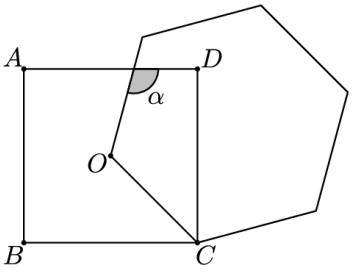
(A) 1 (B) 2 (C) 3 (D) 6 (E) 12

8. John has a number of equally big light and dark cubes. He starts with a dark cube which he places on the table. Now, there are five faces of the cube visible. In his second step, he covers all visible faces of this cube by adding five light cubes as shown. Now, he wants to add dark cubes again so that no light surfaces are visible. What is the minimum number of dark cubes he will require?



(A) 8 (B) 9 (C) 10 (D) 13 (E) 19

9. We draw a square with vertices *A*, *B*, *C*, *D* as shown, and a regular hexagon with side *OC*, where *O* is the centre of the square. How big is angle α?



(A) 105° (B) 110° (C) 115° (D) 120° (E) 125°

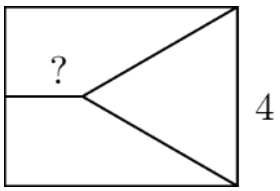
10. Ardal fences a rectangular plot of land. The fence is 40 m long. The lengths of all sides of the rectangle are prime numbers. What is the biggest possible area of the plot of land?

(A) 84 m² (B) 91 m² (C) 96 m² (D) 99 m² (E) 125 m²

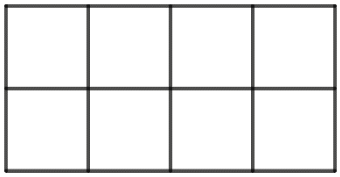
- 4 Point Examples -

11. A palindrome number is a number that is the same read from the front and the back, e.g. 121 or 444.
What is the sum of the digits of the largest three-digit palindrome number that is also a multiple of 6?
(A) 16 (B) 18 (C) 20 (D) 21 (E) 24

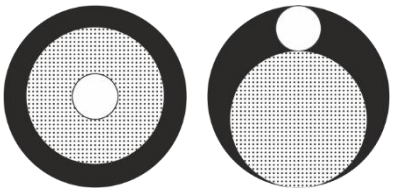
12. A rectangle is split into three pieces with equal area, as shown. One piece is an equilateral triangle with sides of length 4 cm. The other two pieces are trapezoids. How long is the shorter of the parallel sides of the trapezoid?
(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) $2\sqrt{2}$ (D) 3 (E) $2\sqrt{3}$



13. Jelena fills the 2×4-table shown with the letters *A, B, C* and *D*. She wants to make sure that each letter appears exactly once in each row and in each of the three 2×2-squares. How many ways are there for her to do this?
(A) 12 (B) 24 (C) 48 (D) 96 (E) 198



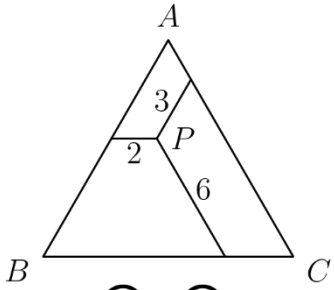
14. Sanjay has three differently coloured circles. First he places them on top of each other as shown in ‚Figur 1'. Then he moves them so that they touch each other pairwise as shown in 'Figur 2'. In Figur 1 the visible black area is seven times as big as the area of the white circle. What is the ratio of the visible black areas in Figur 1 and Figur 2?
(A) 3 : 1 (B) 4 : 3 (C) 6 : 5 (D) 7 : 6 (E) 9 : 7



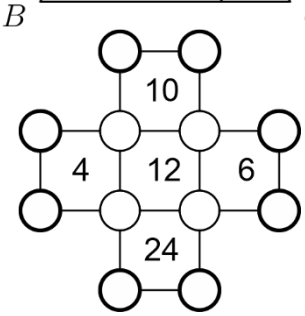
Figur 1 Figur 2

15. The daughter of Mary's daughter was born today. In two years' time the product of Mary's age, her daughter's age and her grand-daughter's age will be exactly 2024. Each of the three ages will then be an even number. How old is Mary today?
(A) 42 (B) 44 (C) 46 (D) 48 (E) 50

16. A point P is chosen inside an equilateral triangle ABC. Segments with the shown lengths of 2 m, 3 m, and 6 m are then drawn parallel to the sides of the triangles, as shown.
What is the perimeter of the triangle?
(A) 22 m (B) 26 m (C) 33 m (D) 39 m (E) 44 m

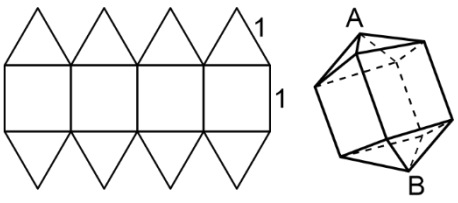


17. A number is written into each of the twelve circles shown. The numbers in the squares state the product of the four numbers in the vertices of the squares. What is the product of the numbers in the eight bold circles?
(A) 20 (B) 40 (C) 80 (D) 120 (E) 480



18. Jean-Philippe has n^3 cubes of equal size. He uses them to form one big cube and paints its surface. The number of small cubes with exactly one painted face is then the same as the number of small cubes with no painted face.
What is the value of *n*?
(A) 4 (B) 6 (C) 7 (D) 8 (E) 10

19. Otis builds the net of a solid using a combination of squares and triangles as shown. All sides of the squares and the triangles have side length 1. He proceeds to fold the net to form the solid shown.
What is the distance from *A* to *B*?
(A) $1+\sqrt{2}$ (B) $1+\sqrt{3}$ (C) $2\sqrt{2}$ (D) $\sqrt{5}$ (E) $\frac{5}{2}$



20. Vlado has participated in 31 cross-country races in the last five years. In the first year, he participated in the smallest number of races and he then successively increased the number of races each year. In the fifth year, he participated in three times as many races as in the first year.
How many races did he participate in in the fourth year?
(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

- 5 Point Examples -

21. How many **integers** *k* have the property that *k+6* is a multiple of *k-6*?
(A) 0 (B) 4 (C) 6 (D) 8 (E) 12

22. There are four bowls with sweets on a table.
The number of sweets in the first bowl is equal to the number of bowls with one sweet.
The number of sweets in the second bowl is equal to the number of bowls with two sweets.
The number of sweets in the third bowl is equal to the number of bowls with three sweets.
The number of sweets in the fourth bowl is equal to the number of bowls with **no** sweets.
How many sweets are in the bowls in total?
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

23. Cristina has 12 cards numbered from 1 to 12. She places eight of them in a circle so that the sum of any two adjacent numbers is a multiple of 3. Which numbers does Christina **not** use?
(A) 1, 5, 9, 12 (B) 3, 5, 7, 9 (C) 1, 2, 11, 12 (D) 5, 6, 7, 8 (E) 3, 6, 9, 12

24. Carl tells the truth on one day, lies on the next, tells the truth again the day after, etc. On one day he made exactly four of the following five statements.
Which statement can he not have made on that day?
A) 2024 is divisible by 11.
B) Tomorrow is Saturday.
C) I lied yesterday and I will lie tomorrow.
D) Yesterday was Wednesday.
E) I am telling the truth today and I will tell the truth tomorrow.

25. The sum of the digits of *N* is three times the sum of the digits of *N*+1.
What is the smallest possible sum of the digits of *N*?
(A) 3 (B) 9 (C) 12 (D) 15 (E) 27

26. Jill has some black and some white unit cubes. She uses 27 of them to build a $3\times3\times3$ cube. She wants exactly one third of the surface to be black.
If *A* is the smallest possible number of black cubes that she can use and *B* the biggest possible number, what is the value of *B* – *A*?
(A) 1 (B) 3 (C) 6 (D) 7 (E) 9

27. Ann rolled an ordinary die 24 times. All numbers from 1 to 6 were rolled at least once. The number 1 was rolled more often than any other number. Ann then added all numbers that were rolled. What is the biggest number she could have obtained in this way?
(A) 83 (B) 84 (C) 89 (D) 90 (E) 100

28. Olga was walking in the park. For half the time, she walked with a speed of 2 km/h. For half the distance, she walked with a speed of 3 km/h. For the remaining time, she walked with a speed of 4 km/h. Which fraction of the time did she walk with 4 km/h?
(A) $\frac{1}{14}$ (B) $\frac{1}{12}$ (C) $\frac{1}{7}$ (D) $\frac{1}{5}$ (E) $\frac{1}{4}$

29. 20 points are distributed equally along a circle. How many of the segments connecting two of those points are longer than the radius of the circle but shorter than the diameter of the circle?
(A) 90 (B) 100 (C) 120 (D) 140 (E) 160

30. Consider *n* different straight lines in a plane, labelled as $\ell_1,\ldots,\ell_n$. The straight line ℓ_1 intersects 5 other straight lines, the straight line ℓ_2 intersects 9 other straight lines and the line ℓ_3 intersects 11 other straight lines. Which of the following numbers is a possible value of *n*?
(A) 11 (B) 12 (C) 13 (D) 14 (E) 15