KÄNGURU DER MATHEMATIK 2024
21. 3. 2024

Level: Student, Grades 11–13

Full name:	
School:	
Class:	

Time: 75 min.
30 starting points
each correct answer to questions 1. – 10.: 3 points
each correct answer to questions 11. – 20.: 4 points
each correct answer to questions 21. – 30.: 5 points
each questions left unanswered: 0 points
each incorrect answer: minus ¼ of the points for the question

Please write the letter (A, B, C, D, E) of the correct answer in the square under the question number (1 bis 30). Write clearly and carefully!

1	2	3	4	5	6	7	8	9	10

11	12	13	14	15	16	17	18	19	20

21	22	23	24	25	26	27	28	29	30

Zustimmungserklärung zur Datenverarbeitung für den österreichischen Wettbewerb „Känguru der Mathematik“

☐Ich stimme zu, dass meine angeführten personenbezogenen Daten (Vor- und Zuname, Klasse, Schulstufe, Schulstandort und Schulart) zum Zweck der Organisation und Durchführung des Wettbewerbs, der Auswertung der Wettbewerbsergebnisse (Ermitteln der erreichten Punkte und Prozentzahlen), des Erstellens von schulweiten Reihungen, sowie zur Erstellung und Veröffentlichung der Siegerlisten auf unserer Vereinshomepage (sofern mindestens 50 % der zu erreichenden Punktezahl erlangt werden bzw. ich unter den besten 10 einer Kategorie liege) verwendet werden dürfen.

Betroffenenrechte
Die Verwendung dieser Daten ist bis 31. Dezember des 2. Folgejahres gestattet. Nach diesem 31. Dezember werden Vor- und Zuname, die Klasse und der Schulstandort gelöscht, wobei dieser durch die Angabe des Bundeslandes ersetzt wird. Die Verwendung der auf diese Art anonymisierten Daten ist nur mehr für statistische Zwecke auf der Grundlage der DSGVO erlaubt.

Ich habe ein Recht auf Auskunft über meine gespeicherten personenbezogenen Daten, sowie das Recht auf Berichtigung, Datenübertragung, Widerspruch, Einschränkung der Bearbeitung sowie Sperrung oder Löschung unrichtig verarbeiteter Daten.

Ich kann die erteilte Einwilligung jederzeit auf der Homepage des Vereines Känguru der Mathematik unter www.kaenguru.at mittels des dafür bereitgestellten Formulars mit Wirkung für die Zukunft widerrufen (Art. 21 Abs. 1 DSGVO).
Ein Widerruf hat zur Folge, dass die personenbezogenen Daten nach gegenseitiger Rücksprache innerhalb von 31 Tagen gelöscht werden.
Durch den Widerruf wird die Rechtmäßigkeit der aufgrund der Einwilligung bis zum Widerruf erfolgten Verarbeitung nicht berührt. (Art. 7 Abs. 2 DSGVO)

Ort, Datum Unterschrift

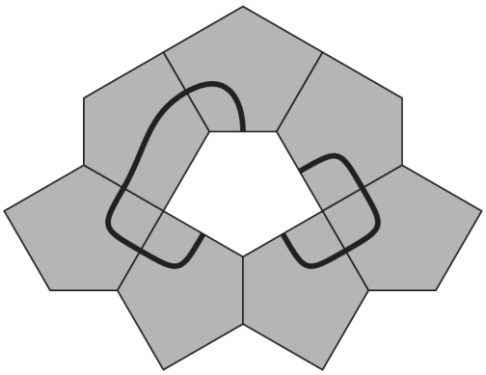
Information über den Känguruwettbewerb: www.kaenguru.at
Wenn du mehr in dieser Richtung machen möchtest,
gibt es die Österreichische Mathematikolympiade.
Infos unter: www.oemo.at

Känguru der Mathematik 2024
Level Student (Schulstufe 11, 12 and 13)
Austria – 21. 3. 2024

- 3 Point Examples -

1. The angles in a triangle are in the ratio $1:3:5$. What is the biggest of those angles?
(A) 80° (B) 90° (C) 100° (D) 110° (E) 120°

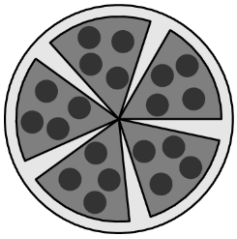
2. A tile pattern is made up of a number of identical irregular pentagons. Which of the following tiles fits into the hole in such a way that a closed curve is formed?



(A)  (B)  (C) 

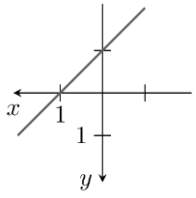
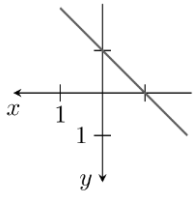
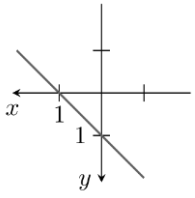
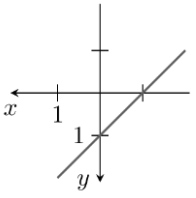
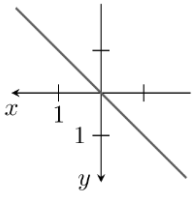
(D)  (E) 

3. Which of the following numbers is two less than a multiple of ten, two more than a square number and two times a prime number?
(A) 78 (B) 58 (C) 38 (D) 18 (E) 6



4. A kangaroo cuts a pizza into 6 pieces of equal size. After it has eaten one piece, it rearranges the remaining pieces, so that the gaps between the pieces are all equally big. What is the angle in each gap?
(A) 5° (B) 8° (C) 9° (D) 10° (E) 12°

5. Julia has the strange habit of drawing the *xy*-plane with the positive directions of the coordinate axes pointing to the left and downwards.
What does the graph of the equation y = x + 1 look like in Julias co-ordinate system?

(A)  (B)  (C)  (D)  (E) 

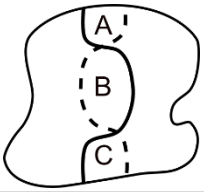
6. Kaito has manipulated a die. The probabilities of rolling a 2, 3, 4 or 5 are still $\frac{1}{6}$ each but the probability of rolling a 6 is now twice the probability of rolling a 1.
What is the probability of rolling a 6?
(A) $\frac{1}{4}$ (B) $\frac{1}{6}$ (C) $\frac{7}{36}$ (D) $\frac{2}{9}$ (E) $\frac{5}{18}$

7. Which of the following expressions has the same value as $16^{15}+16^{15}+16^{15}+16^{15}$?
(A) 4^{31} (B) 4^{38} (C) 4^{60} (D) 4^{120} (E) 4^{122}

8. There are 6 coins on a table, each with heads facing upwards. On each move we turn over exactly 4 of the coins.
What is the minimum number of moves we must make, such that all coins are left with heads facing downwards?
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

9. Noah starts with the number 1 and multiplies it with either 6 or 10. He then multiplies the result again by either 6 or 10. He repeats this process several times.
Which of the following numbers can he **not** obtain in this way?
(A) $2^{100}\cdot 3^{20}\cdot 5^{80}$ (B) $2^{90}\cdot 3^{30}\cdot 5^{80}$ (C) $2^{90}\cdot 3^{20}\cdot 5^{70}$ (D) $2^{110}\cdot 3^{80}\cdot 5^{30}$ (E) $2^{50}\cdot 5^{50}$

10. There are black and dashed paths in a park. Both paths divide the area of the park exactly in half. Which of the following statements about the areas of the sections A, B and C is definitely correct?

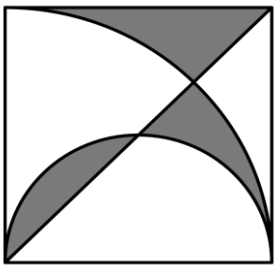


(A) $A=C$ (B) $B=A+C$ (C) $B=\frac{1}{2}\cdot(A+C)$ (D) $B=\frac{2}{3}\cdot(A+C)$ (E) $B=\frac{3}{5}\cdot(A+C)$

- 4 Point Examples -

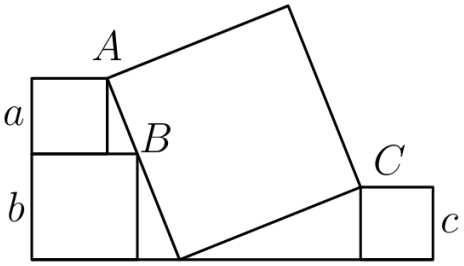
11. John has black and white unit cubes and wants to use 27 of them to build a $3\times3\times3$ cube. He wants to make sure that the surface is exactly half white and half black.
What is the minimum number of black cubes that he needs?
(A) 14 (B) 13 (C) 12 (D) 11 (E) another number

12. In a square with side length 6 a diagonal, a semi-circle and a quarter circle are drawn as shown.
What is the area of the grey region?



(A) 9 (B) $3\cdot\pi$ (C) $6\cdot\pi-9$ (D) $\frac{10}{3}\cdot\pi$ (E) 12

13. The diagram shows four squares with the entire configuration resting on a horizontal straight line. The smaller squares have side lengths *a*, *b* and *c*. The vertices *A* and *C* of two small squares coincide with diagonally opposite vertices of the big square. The vertex *B* of the third small square lies on a side of the big square. Which of the following expressions is equal to the side length of the big square?

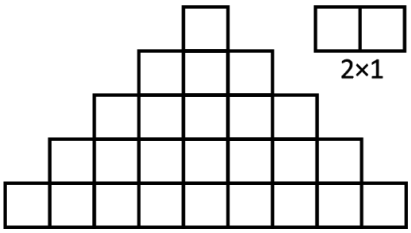


(A) $\frac{1}{2}\cdot(a+b+c)$ (B) $\sqrt{a^2+b^2+c^2}$ (C) $\sqrt{(a+b)^2+c^2}$
(D) $\sqrt{(b-a)^2+c^2}$ (E) $\sqrt{a^2+ab+b^2+c^2}$

14. We are given two positive numbers *x* and *y* with $x < y$. Which of the following expressions has the biggest value?

(A) $\frac{x+3y}{4}$ (B) $\frac{x+2y}{3}$ (C) $\frac{x+y}{2}$ (D) $\frac{2x+y}{3}$ (E) $\frac{3x+y}{4}$

15. The following shape is composed of identical squares. What is the maximum number of 2×1 -dominoes that can be placed on the shape if each covers exactly two squares? The dominoes can be placed horizontally or vertically and are not allowed to cover each other.



(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

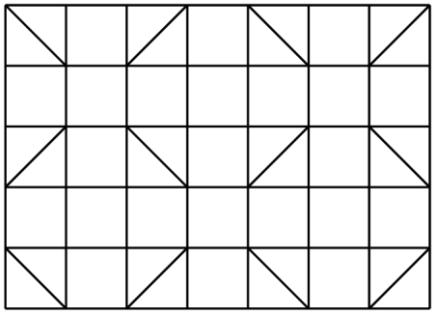
16. How many three-digit numbers are there that contain at least one of the digits 1, 2 or 3?
(A) 27 (B) 147 (C) 441 (D) 557 (E) 606

17. A teacher writes the 7 digits shown on the board. He asks a student to insert some multiplication signs (x) in such a way that the product of the resulting numbers (possibly with multiple digits) has the value 2024. How many multiplication signs must be inserted?

1 1 2 2 2 2 3

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

18. A beaver wants to colour the squares and triangles in the pattern so that adjacent cells are never the same colour, even if they only touch each other in one corner. What is the minimum number of colours he needs?



(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

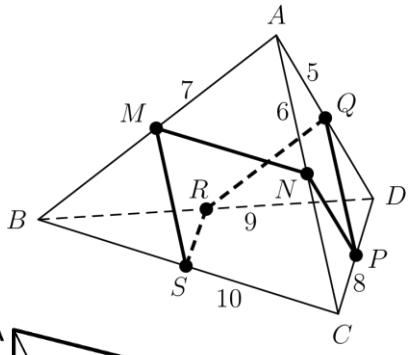
19. We know of a positive integer *n* that exactly one of the following statements is true. Which is the true statement?
(A) *n* is divisible by 3. (B) *n* is divisible by 6. (C) *n* is odd.
(D) $n = 2$ (E) *n* is prime.

20. Two candles of equal length are lit at the same time. One candle will burn down completely in 4 hours, the other in 5 hours. Both burn at a constant rate. How many hours do they have to burn until one candle is exactly 3 times as long as the other?

(A) $\frac{40}{11}$ (B) $\frac{45}{12}$ (C) $\frac{63}{20}$ (D) 3 (E) $\frac{47}{14}$

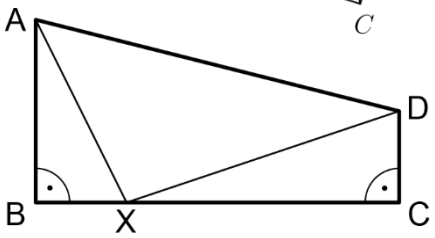
- 5 Point Examples -

21. A three-sided pyramid has edges with side lengths 5, 6, 7, 8, 9 and 10. The points *M*, *N*, *P*, *Q*, *R* and *S* are the midpoints of the edges, as shown in the diagram. What is the total length of the closed polyline *MNPQRSM*?



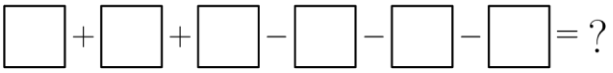
(A) 19 (B) 20 (C) 21 (D) 22 (E) 23

22. A quadrilateral *ABCD* has two right angles in the vertices *B* and *C*.
It is known that *AB* = 4, *BC* = 8 and *CD* = 2. What is the smallest possible value of *AX* + *DX*, if *X* is a point on the segment *BC*?



(A) $9\sqrt{2}$ (B) 12 (C) 13 (D) 10 (E) another value

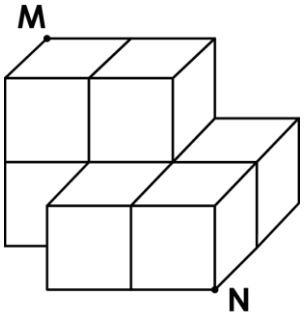
23. We have 6 cards and there is one number written on each side of each card. The pairs of numbers on the cards are (5,12), (3,11), (0,16), (7,8), (4,14) and (9,10). The cards can be placed in the empty squares in any order with any side up.



What is the smallest possible result of the calculation?

(A) -23 (B) -24 (C) -25 (D) -26 (E) -27

24. I have a four-digit number $N = \overline{pqrs}$. If I place a decimal point between the digits *q* and *r*, I obtain the number $\overline{pq.rs}$. This is exactly the average of the two numbers $\overline{pq}$ and $\overline{rs}$.
What is the sum of the digits of *N*?



(A) 14 (B) 18 (C) 21 (D) 25 (E) 27

25. The diagram shows an object composed of 7 cubes with edge length 2. How long is the shortest path from *M* to *N* on the surface of the object?

(A) 10 (B) $2\sqrt{5}+4\sqrt{2}$ (C) $4\sqrt{5}+2$ (D) $4+2\sqrt{2}+2\sqrt{5}$ (E) 14

26. Sylvia has several fair 12-sided dice, each with the numbers 1 to 12 written on their faces.
If she rolls all the dice simultaneously, the probability of rolling exactly one 12 is equally to the probability of not rolling a 12 at all. How many dice does Sylvia have?

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

27. It is known that the statements $2^x = 3$, $2^y = 7$ and $6^z = 7$ are true. Which of the following relationships is therefore correct?

(A) $z = \frac{y}{1+x}$ (B) $z = \frac{x}{y}+1$ (C) $z = \frac{y}{x}-1$ (D) $z = \frac{x}{y-1}$ (E) $z = y-\frac{1}{x}$

28. A function $f: R \to R$ fulfils the condition $f(20-x) = f(22+x)$ for all real numbers *x*. It is known that *f* has exactly two real zeros. What is the sum of the two zeros?

(A) -1 (B) 20 (C) 21 (D) 22 (E) another number

29. A special four-digit number $\overline{abcd}$ fulfils the equation $\overline{abcd} = a^a + b^b + c^c + d^d$. What is the value of a?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

30. A game board is composed of 8 squares on which we want to stack coins. Initially, all squares are empty. On each turn we choose four adjacent squares and place one coin on each of those squares. The numbers show how high the stacks are. Unfortunately, the table wobbled and five of the stacks fell over.

★	30	42	★	★	36	?	★

How many coins were on the field indicated with a question mark before the stack fell?

(A) 24 (B) 30 (C) 36 (D) 48 (E) another number